

SINP-TNP/06-21

hep-ph/0608131

# Radiative neutrino decay and CP-violation in $R$ -parity violating supersymmetry

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## Abstract

We calculate the radiative decay amplitude for Majorana neutrinos in trilinear  $R$ -parity violating supersymmetric framework. Our results make no assumption regarding the masses and mixings of fermions and sfermions. The results obtained are exemplary for generic models with loop-generated neutrino masses. Comparison of this amplitude with the neutrino mass matrix shows that the two provide *independent* probes of CP-violating phases.

## 1 Introduction

In the conventional minimal supersymmetric standard model (MSSM) without right-handed neutrinos and with conserved  $R$ -parity it is not possible to write renormalizable operators which generate neutrino masses. However, conservation of lepton and baryon numbers leading to exact  $R$ -parity is an *ad-hoc* symmetry in the MSSM, and there is no deep underlying principle as to why such a discrete symmetry will remain conserved. Once we admit  $R$ -parity violating (RPV) couplings [1] in the superpotential, neutrino masses are generated through diagrams involving RPV couplings. These are Majorana masses which result as a consequence of broken lepton number(s). There are two kinds of explicit RPV couplings: bilinear and trilinear. The bilinear ones induce sneutrino vacuum expectation values, allowing neutrinos to mix with the neutralinos, and in this mechanism only one physical neutrino obtains mass at the tree level [2]. Trilinear Yukawa type couplings generate a complete neutrino mass matrix through one-loop self-energy graphs

arXiv:hep-ph/0608131 v2 8 Sep 2006

[3]. Obviously, one can take a combination of bilinear and trilinear couplings, along with possible soft terms, for a complete analysis [4]. Here we concentrate only on the trilinear superpotential couplings. Importantly, the diagrams that generate neutrino masses also generate dipole moments by insertion of photons to the internal lines. Clearly, in the absence of Dirac-type operators, such dipole moments are transitional.

The connection between neutrino Majorana masses and transition moments have been studied in the past [5, 6] and predictions have been made using the constraints on trilinear RPV couplings placed from the data on neutrino masses and mixings [7] (compare the present update in [8]). However, as regards the analytic formulation of dipole moments, two crucial things were still missing. One of these is a general analysis with the inclusion of fermion and sfermion flavor mixings. The other is a careful handling of the phases from different sources that enter the analysis. We try to address these two issues in the present paper. In doing so, we ask the following question: how different is the combination of phases that appears in the neutrino radiative decay from the one that appears in the neutrinoless double beta decay? We keep the formalism at a very general level. Thus our results can be applied, with appropriate changes in notation, to a general class of models [9] involving Majorana neutrinos, although for the sake of definiteness we choose the supersymmetric RPV model, for which the lepton number violating part of the RPV superpotential can be written as

$$\mathcal{W} = \frac{1}{2} \sum_{ijk} \lambda_{ijk} L_i L_j E_k^c + \sum_{ijk} \lambda'_{ijk} L_i Q_j D_k^c. \quad (1)$$

Here  $L_i$  denotes the doublet leptonic superfield and  $Q_i$  is the doublet quark superfield, whereas  $E_i^c$  and  $D_i^c$  denote SU(2)-singlet superfields which contain the left-chiral components of charged antileptons and down-type antiquarks. Stringent upper limits exist on all these couplings from different experiments [10, 11].

## 2 Feynman rules with fermion and sfermion mixings

Without any loss of generality, we can take the quark superfields in a basis such that the up-type quark mass matrix will come out to be diagonal. Also, the lepton superfields will be defined in a basis where the charged lepton mass matrices will come out to be diagonal. In addition, we can take the singlet fields in a way that they will not have to be diagonalized further. This will be called the *flavor basis*.

The mass matrices for neutrinos and the  $d$ -type quarks will be non-diagonal in this basis. We define the mass eigenstates as follows:

$$\begin{aligned} \nu_{iL} &= \sum_{\alpha} U_{i\alpha} \nu_{\alpha L}, \\ d_{iL} &= \sum_a V_{ia} d_{aL}, \end{aligned} \quad (2)$$

where in each case, the states on the left side are the flavor states, and the states on the right are mass eigenstates. The matrices  $U$  and  $V$  are respectively the PMNS and the

CKM mixing matrices for leptons and quarks. The squark fields will also mix in general. We will denote the superpartners of the left-chiral quark fields  $d_L$  by  $\tilde{d}$ , without the subscript  $L$  since the scalar field does not carry any chirality. Similarly  $d_L^c$ , the conjugate of right-chiral quark fields  $d_R$ , have superpartners which will be denoted by  $\tilde{d}^c$ . In the mass matrix, the fields  $\tilde{d}$  will mix with the fields  $\tilde{d}^{c\dagger}$ . For three generations, there will be then six eigenstates for down squarks. These will be denoted by  $\Delta_A$ , where

$$\begin{aligned}\tilde{d}_k &= \sum_A K_{kA} \Delta_A, \\ \tilde{d}_k^{c\dagger} &= \sum_A \hat{K}_{kA} \Delta_A.\end{aligned}\tag{3}$$

Here, for  $n$  fermion generations, both  $K$  and  $\hat{K}$  are  $n \times 2n$  matrices, which are respectively the first and last  $n$  rows of a  $2n \times 2n$  unitary mixing matrix  $\mathbb{K}$  of the down squark sector.

From the superpotential given in Eq. (1), when we derive the Lagrangian containing ordinary fields, any cubic term in the superpotential produces various kinds of Yukawa couplings. We are interested in processes with external neutrino lines, so we seek for terms where the neutrino field (and not the sneutrino field) appears. For the time being, let us deal with only the second term, i.e., the  $\lambda'$  term. We shall comment on the inclusion of the  $\lambda$  term later. The relevant Yukawa couplings are then:

$$\mathcal{L}'_Y = \sum_{ijk} \lambda'_{ijk} \nu_{iL}^\top C^{-1} d_{jL} \tilde{d}_k^c + \sum_{ijk} \lambda'_{ijk} \nu_{iL}^\top C^{-1} d_{kL}^c \tilde{d}_j + \text{h.c.}\tag{4}$$

Using Eq. (2), this can be rewritten in terms of mass eigenstates in the form

$$\mathcal{L}'_Y = \sum_{\alpha a A} \hat{\Lambda}'_{\alpha a A} \nu_\alpha^\top C^{-1} P_L d_a \Delta_A^\dagger + \sum_{\alpha a A} \Lambda'_{\alpha a A} \nu_\alpha^\top C^{-1} P_L d_a^c \Delta_A + \text{h.c.},\tag{5}$$

where

$$\begin{aligned}\hat{\Lambda}'_{\alpha a A} &= \sum_{ijk} \lambda'_{ijk} U_{i\alpha} V_{ja} \hat{K}_{kA}^*, \\ \Lambda'_{\alpha a A} &= \sum_{ijk} \lambda'_{ijk} U_{i\alpha} \delta_{ka} K_{jA}.\end{aligned}\tag{6}$$

The appearance of the Kronecker delta in the latter formula is a reminder that for singlet fields, the flavor indices and the mass indices are interchangeable. In Eq. (5), we have also used the notation for chiral projection operators that will be used henceforth:

$$P_L = \frac{1}{2}(1 - \gamma_5), \quad P_R = \frac{1}{2}(1 + \gamma_5).\tag{7}$$

Contributions to radiative neutrino decay occur at the 1-loop level through diagrams such as those shown in Fig. 1. If the matrix  $\mathbb{K}$  is block-diagonal, i.e., there is no mixing between the fields  $\tilde{d}$  and the fields  $\tilde{d}^{c\dagger}$  in the mass matrix, contributions will be proportional

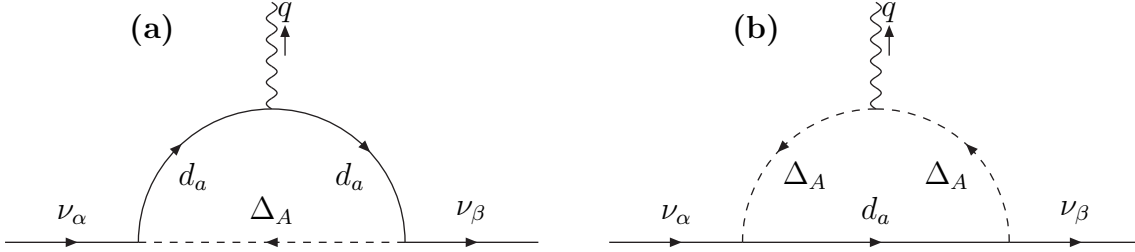


Figure 1: 1-loop diagrams for the neutrino photon vertex. All fields shown are mass eigenstates. The photon attaches to the quark in diagram (a), and to the squark in (b).

to the external neutrino masses and should therefore be small. We will be interested in the more general case where  $\mathbb{K}$  is not necessarily block-diagonal. Diagrams for this general case are shown in Fig. 1. As we will see, in this case the radiative neutrino decay amplitude will be proportional to the masses of the down-type quarks.

Before calculating the amplitude, let us put Eq. (5) in a more conventional form. We have used the symbol  $d^c$ . This is the conjugate of the  $d$ -quark field. For any fermion field  $\psi$ , the Lorentz-covariant conjugate can be defined as

$$\psi^c = \gamma_0 C \psi^*. \quad (8)$$

It is then easy to see that

$$\psi^\top C^{-1} = \psi^\top C^\dagger = \left(C \psi^*\right)^\dagger = \left(\gamma_0 \psi^c\right)^\dagger = \overline{\psi^c}. \quad (9)$$

Antisymmetry of the matrix  $C$  follows from the relation  $(\psi^c)^c = \psi$ . Using this as well as the anticommutation of fermion fields, we can write

$$\nu_\alpha^\top C^{-1} \mathbf{P}_L d_a^c = d_a^{c\top} C^{-1} \mathbf{P}_L \nu_\alpha = \bar{d}_a \mathbf{P}_L \nu_\alpha, \quad (10)$$

using Eq. (9) in the last step.

Further, for Majorana neutrino fields, the conjugate field is the same as the original fields, apart from possibly a phase factor:

$$\nu_\alpha = \eta_\alpha \nu_\alpha^c. \quad (11)$$

Thus, for Majorana neutrino fields, we obtain

$$\nu_\alpha^\top C^{-1} = \overline{\nu^c}_\alpha = \eta_\alpha \bar{\nu}_\alpha. \quad (12)$$

Putting Eqs. (10) and (12) back into Eq. (5), we obtain

$$\mathcal{L}'_Y = \sum_{\alpha a A} \left[ \bar{\nu}_\alpha \left( \eta_\alpha \hat{\Lambda}'_{\alpha a A} \mathbf{P}_L + \Lambda'^*_{\alpha a A} \mathbf{P}_R \right) d_a \Delta_A^\dagger + \bar{d}_a \left( \eta_\alpha^* \hat{\Lambda}'^*_{\alpha a A} \mathbf{P}_R + \Lambda'_{\alpha a A} \mathbf{P}_L \right) \nu_\alpha \Delta_A \right], \quad (13)$$

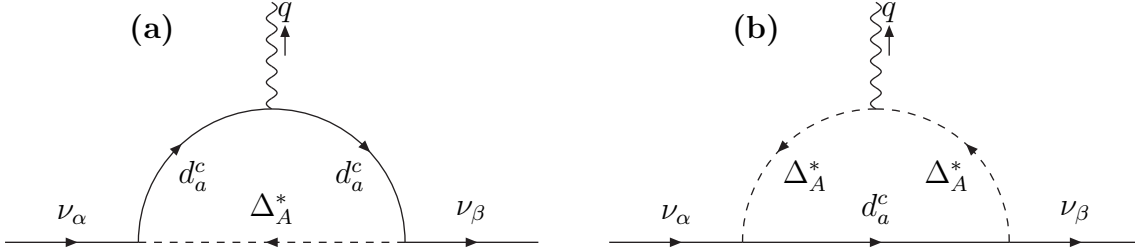


Figure 2: Same as in Fig. 1, except the internal lines are now conjugated.

where we have written the hermitian conjugate terms explicitly. By using Eqs. (8) and (11), these interactions can also be written as

$$\mathcal{L}'_Y = \sum_{\alpha a A} \left[ \bar{d}_a^c \left( \hat{\Lambda}'_{\alpha a A} \mathbf{P}_L + \eta_\alpha^* \Lambda_{\alpha a A}'^* \mathbf{P}_R \right) \nu_\alpha \Delta_A^\dagger + \bar{\nu}_\alpha \left( \hat{\Lambda}_{\alpha a A}'^* \mathbf{P}_R + \eta_\alpha \Lambda_{\alpha a A}' \mathbf{P}_L \right) d_a^c \Delta_A \right]. \quad (14)$$

This form will be useful when we calculate the diagrams shown in Fig. 2, where the internal particles are conjugated. As is well-known, such conjugated diagrams exist when we deal with Majorana neutrinos [12].

### 3 Radiative neutrino decay amplitude

#### 3.1 Calculation of the vertex function

The vertex function  $\Gamma^\lambda$  will be defined in such a way that the matrix element of the photon vertex with on-shell fermions is given by

$$\mathcal{M} = \bar{u}_\beta(p') \Gamma^\lambda u_\alpha(p) A_\lambda(q). \quad (15)$$

For the diagram in Fig. 1a, the contribution to the vertex function will be denoted by  $\Gamma_{(1a)}^\lambda$ , which can be easily written down, using the vertex Feynman rules from the interaction given in Eq. (13). The result is

$$i\Gamma_{(1a)}^\lambda = \sum_{aA} \int \frac{d^4 k}{(2\pi)^4} i \left( \eta_\beta \hat{\Lambda}'_{\beta a A} \mathbf{P}_L + \Lambda_{\beta a A}'^* \mathbf{P}_R \right) \frac{i(\not{k} - \not{q} + m_{d_a})}{(k - q)^2 - m_{d_a}^2} \\ \times \left( i e_d \gamma^\lambda \right) \frac{i(\not{k} + m_{d_a})}{k^2 - m_{d_a}^2} i \left( \eta_\alpha^* \hat{\Lambda}_{\alpha a A}'^* \mathbf{P}_R + \Lambda_{\alpha a A}' \mathbf{P}_L \right) \frac{i}{(k - p)^2 - \widetilde{M}_{\Delta_A}^2}, \quad (16)$$

where  $e_d$  is the charge of the  $d$ -quark. From a general analysis of the form factors related to the electromagnetic vertices, we know that the radiative transition amplitudes can have only the magnetic and electric dipole form factors. These are helicity flipping terms,

containing an even number of Dirac matrices sandwiched between the spinors. We can separate out such terms and write

$$\begin{aligned}\Gamma_{(1a)}^\lambda &= ie_d \sum_{aA} m_{da} \left( \eta_\beta \hat{\Lambda}'_{\beta aA} \Lambda'_{\alpha aA} \mathbf{P}_L + \eta_\alpha^* \hat{\Lambda}'_{\alpha aA}^* \Lambda'_{\beta aA}^* \mathbf{P}_R \right) \\ &\times \int \frac{d^4 k}{(2\pi)^4} \frac{(\not{k} - \not{q})\gamma^\lambda + \gamma^\lambda \not{k}}{\left[ k^2 - m_{da}^2 \right] \left[ (k - q)^2 - m_{da}^2 \right] \left[ (k - p)^2 - \widetilde{M}_{\Delta_A}^2 \right]}.\end{aligned}\quad (17)$$

Similarly, we can write down the contribution of Fig. 1b. By combining the factors in the denominator of the integrand, we can identify the magnetic and electric dipole terms, which are of the form

$$\Gamma_{(1)}^\lambda = ie_d \sigma^{\lambda\rho} q_\rho \sum_{aA} m_{da} \left( \eta_\beta \hat{\Lambda}'_{\beta aA} \Lambda'_{\alpha aA} \mathbf{P}_L + \eta_\alpha^* \hat{\Lambda}'_{\alpha aA}^* \Lambda'_{\beta aA}^* \mathbf{P}_R \right) J_{aA}. \quad (18)$$

Contributions to  $J_{aA}$  from Fig. 1a and Fig. 1b are given by

$$J_{aA}^{(1a)} = i \int_0^1 dx \int \frac{d^4 k}{(2\pi)^4} \frac{-2(1-x)^2}{\left[ k^2 - x\widetilde{M}_{\Delta_A}^2 - (1-x)m_{da}^2 \right]^3}, \quad (19)$$

$$J_{aA}^{(1b)} = i \int_0^1 dx \int \frac{d^4 k}{(2\pi)^4} \frac{-2x(1-x)}{\left[ k^2 - x\widetilde{M}_{\Delta_A}^2 - (1-x)m_{da}^2 \right]^3}. \quad (20)$$

In writing these integrals, we have neglected neutrino masses in the denominators. Thus

$$J_{aA}^{(1)} = i \int_0^1 dx \int \frac{d^4 k}{(2\pi)^4} \frac{-2(1-x)}{\left[ k^2 - x\widetilde{M}_{\Delta_A}^2 - (1-x)m_{da}^2 \right]^3}. \quad (21)$$

We now come to the conjugated diagrams of Fig. 2, which are related to the diagrams of Fig. 1 through

$$d_a \longrightarrow d_a^c, \quad \Delta_A \longrightarrow \Delta_A^\dagger. \quad (22)$$

It is now convenient to use the Yukawa couplings in the form given in Eq. (14). Comparing this with Eq. (13), we see that the corresponding amplitudes are related by the substitutions

$$\Lambda_{\alpha aA} \longleftrightarrow \hat{\Lambda}_{\alpha aA}, \quad (23)$$

and an overall negative sign because the photon line now attaches to an internal line of opposite charge. So we obtain

$$\Gamma_{(2)}^\lambda = -ie_d \sigma^{\lambda\rho} q_\rho \sum_{aA} m_{da} \left( \eta_\beta \hat{\Lambda}'_{\alpha aA} \Lambda'_{\beta aA} \mathbf{P}_L + \eta_\alpha^* \hat{\Lambda}'_{\beta aA}^* \Lambda'_{\alpha aA}^* \mathbf{P}_R \right) J_{aA}. \quad (24)$$

Combining the contributions of Figs. 1 and 2, we can therefore write the total vertex function  $\Gamma^\lambda$  as

$$\begin{aligned} \Gamma^\lambda = ie_d \sigma^{\lambda\rho} q_\rho \sum_{aA} m_{da} \Big[ \eta_\beta \Big( \widehat{\Lambda}'_{\beta aA} \Lambda'_{\alpha aA} - \widehat{\Lambda}'_{\alpha aA} \Lambda'_{\beta aA} \Big) \text{P}_L \\ + \eta_\alpha^* \Big( \widehat{\Lambda}'^*_{\alpha aA} \Lambda'^*_{\beta aA} - \widehat{\Lambda}'^*_{\beta aA} \Lambda'^*_{\alpha aA} \Big) \text{P}_R \Big] J_{aA}. \end{aligned} \quad (25)$$

Clearly, if  $\alpha = \beta$ , i.e., we are talking about the diagonal vertex, the contribution vanishes, as is required by CPT invariance.

### 3.2 CP-invariant limit

Before proceeding further with the calculation, it would be instructive to check the form of the vertex in the CP-invariant limit. The CP transformation property of a fermion field  $\psi$  is given by

$$(\mathcal{CP})\psi(\vec{x}, t)(\mathcal{CP})^{-1} = \xi^* C \psi^*(-\vec{x}, t), \quad (26)$$

where  $\xi$  can be called the CP phase of the field. For the scalar and pseudoscalar bilinears that occur in the Yukawa coupling of Eq. (13), the effect of CP transformation is then given by

$$\begin{aligned} (\mathcal{CP})\bar{\psi}_1 \psi_2 (\mathcal{CP})^{-1} &= \xi_1 \xi_2^* \bar{\psi}_2 \psi_1, \\ (\mathcal{CP})\bar{\psi}_1 \gamma_5 \psi_2 (\mathcal{CP})^{-1} &= -\xi_1 \xi_2^* \bar{\psi}_2 \gamma_5 \psi_1, \end{aligned} \quad (27)$$

where the space-time points have not been explicitly mentioned. For all Dirac fermion fields, the CP phase can be absorbed into the definition of the antiparticle without any loss of generality. For Majorana neutrinos, however, that cannot be done because we have already defined  $\nu_\alpha^c$  through Eq. (11). In what follows, we will denote the CP phase of the field  $\nu_\alpha$  by  $\xi_\alpha$ .

We now take the CP transformation of the first term on the right side of Eq. (13).

$$(\mathcal{CP})\bar{\nu}_\alpha \left( \eta_\alpha \widehat{\Lambda}'_{\alpha aA} \text{P}_L + \Lambda'^*_{\alpha aA} \text{P}_R \right) d_a \Delta_A^\dagger (\mathcal{CP})^{-1} = \xi_\alpha \bar{d}_a \left( \eta_\alpha \widehat{\Lambda}'_{\alpha aA} \text{P}_R + \Lambda'^*_{\alpha aA} \text{P}_L \right) \nu_\alpha \Delta_A. \quad (28)$$

If the Lagrangian is CP invariant, this CP transform should equal the hermitian conjugate term present in Eq. (13). This imposes the following conditions on the Yukawa couplings:

$$\begin{aligned} \widehat{\Lambda}'^*_{\alpha aA} &= \xi_\alpha \eta_\alpha^2 \widehat{\Lambda}'_{\alpha aA}, \\ \Lambda'^*_{\alpha aA} &= \xi_\alpha^* \Lambda'_{\alpha aA}. \end{aligned} \quad (29)$$

If we put these conditions back into Eq. (25), we find that the expression for the vertex function reduces to the form

$$\Gamma^\lambda = ie_d \sigma^{\lambda\rho} q_\rho \eta_\beta \sum_{aA} m_{da} \left[ \widehat{\Lambda}'_{\beta aA} \Lambda'_{\alpha aA} \left( \text{P}_L - \zeta_{\alpha\beta}^* \text{P}_R \right) + \widehat{\Lambda}'_{\alpha aA} \Lambda'_{\beta aA} \left( \zeta_{\alpha\beta} \text{P}_R - \text{P}_L \right) \right] J_{aA}, \quad (30)$$

where

$$\zeta_{\alpha\beta} = \xi_\alpha \eta_\alpha \xi_\beta^* \eta_\beta^* = \frac{\xi_\alpha \eta_\alpha}{\xi_\beta \eta_\beta}. \quad (31)$$

For Majorana fields, the quantity  $\xi\eta$  equals the CP eigenvalue of the 1-particle states [13, 14, 15], so that  $\zeta_{\alpha\beta}$  denotes the relative CP eigenvalues of the two Majorana neutrinos involved in the process. Thus, Eq. (30) shows that in the CP-conserving case, the transition amplitude is purely electric dipole type if the CP eigenvalues are the same for the two neutrinos, and purely magnetic dipole type if the CP eigenvalues are opposite. This is expected on quite general grounds [16].

### 3.3 CP-violating scenario

We now go back to the general case, where the couplings do not necessarily obey Eq. (29) and therefore the Lagrangian does not conserve CP. For such a case, we cannot use the vertex of Eq. (30). Rather, we need to go back to the general form of the vertex given in Eq. (25). In fact, this expression can be written in a simpler form, noting the fact that the neutrino mixing matrix  $U$  and the phases  $\eta_\alpha$  are interconnected. In general, any mass matrix  $M$  can be diagonalized by a bi-unitary transformation involving two unitary matrices  $U$  and  $U'$ :

$$U'^\dagger M U = D, \quad (32)$$

where  $D$  is a diagonal matrix whose diagonal elements are the mass eigenvalues. Eq. (32) implies  $U^\top M^\top U'^* = D^\top$ . For Majorana neutrinos, the mass matrix is symmetric. So, using  $M = M^\top$  and  $D = D^\top$ , we see that it is possible to choose  $U' = U^*$ . Thus the bi-unitary transformation reduces to

$$U^\top M U = D, \quad (33)$$

We will adopt this convention in what follows. This automatically implies [17]

$$\eta_\alpha = 1 \quad \text{for each } \alpha. \quad (34)$$

Of course, other choices are possible [17], but they all lead to same physical results. Therefore, the choice of Eq. (34) does not mean any loss of generality.

To proceed, we make some simplifying assumptions. First, motivated by the fact that quark mixing is small, we will neglect it altogether and use the unit matrix in place of  $V$ . As mentioned earlier, the  $\tilde{d}\tilde{d}^{c\dagger}$  mixing is crucial for the terms that we have been considering in Figs. 1 and 2, viz., terms having a factor of a down-type quark mass, let us assume that these are the only kind of mixings in the squark sector mass matrix. Further we assume that there is no intergenerational mixing in this sector, which means that the  $\tilde{d}$  squark of a certain generation mixes only with  $\tilde{d}^{c\dagger}$  of the same generation. Then the mixing matrix  $\mathbb{K}$  in the squark sector is real, and is given by

$$\mathbb{K} \equiv \begin{pmatrix} K \\ \hat{K} \end{pmatrix} = \begin{pmatrix} C & S \\ -S & C \end{pmatrix}, \quad (35)$$



where in the final form, each entry denotes a  $n \times n$  diagonal matrix, where  $n$  is the number of generations, and

$$C \equiv \text{diag} (\cos \theta_1, \cos \theta_2, \dots), \quad S \equiv \text{diag} (\sin \theta_1, \sin \theta_2, \dots). \quad (36)$$

More explicitly, we can write the elements of  $K$  and  $\widehat{K}$  as

$$\begin{aligned} K_{kA} &= \cos \theta_k \delta_{k,A} + \sin \theta_k \delta_{k+n,A} \\ \widehat{K}_{kA} &= -\sin \theta_k \delta_{k,A} + \cos \theta_k \delta_{k+n,A}. \end{aligned} \quad (37)$$

It should be recalled, from the definition of Eq. (3), that the index  $A$  runs over  $2n$  values for  $n$  generations.

The only intergenerational mixing in this picture is in the neutrino sector, which contains the phases of our concern. The vertex of Eq. (25) can then be written as:

$$\Gamma^\lambda = ie_d \sigma^{\lambda\rho} q_\rho \left[ F \mathbf{P}_L + F^* \mathbf{P}_R \right], \quad (38)$$

where

$$F = \sum_{jk} m_{d_j} B_{jk} \cos \theta_k \sin \theta_k \left( J_{j,k+n} - J_{jk} \right), \quad (39)$$

with  $B_{jk}$  defined by

$$B_{jk} = \sum_{ii'} \left( U_{i'\alpha} U_{i\beta} - U_{i\alpha} U_{i'\beta} \right) \lambda'_{ijk} \lambda'_{i'kj}. \quad (40)$$

Apparently, there is one big difference between this form of the vertex function and the expressions derived in an earlier work on the subject [7]. The expression in Ref. [7] contains two explicit factors of down quark masses. It should be emphasized that the extra factor, compared to our Eq. (39), appears from an extra assumption. To appreciate this, consider what happens if all down-type squarks were really degenerate. The integrals defined in Eqs. (19) and (20) would have been independent of the squark index  $A$  in that case. Looking at Eq. (6), we find that in Eq. (25), the sum over  $A$  would have involved only the combination

$$\sum_A \widehat{K}_{kA}^* K_{jA} = 0, \quad (41)$$

because of the orthogonality of two different rows of the unitary matrix  $\mathbb{K}$ . The result implies that the vertex has to involve only differences of squark masses. These differences have to be small from various phenomenological requirements, and one often assumes that the differences are proportional to the corresponding quark masses. In particular, Eq. (39) shows that in absence of intergenerational mixing, the contribution from the two

squarks of a single generation vanishes if they are degenerate. Customarily, one assumes that the mass squared differences of the two squarks in the same generation is of the form

$$\Delta\widetilde{M}^2 = \widetilde{M}m_q, \quad (42)$$

where  $m_q$  is the mass of the quark in the same generation and  $\widetilde{M}$  is the average mass of the two squarks. It is this assumption which produced the extra factor of quark mass in the expression of Ref. [7]. We will carry on our discussion without this specific form for the squark mass differences. The phenomenological consequences of the decay rate, and the resulting bounds obtained thereby, have been discussed in earlier papers [7, 8]. In our discussion, we highlight CP violating features of this amplitude.

## 4 Exploring phenomenological consequences

### 4.1 The mass matrix

Since the mixing matrix  $U$  is present in the vertex function through the combination of Eq. (40), the radiative decay amplitudes will clearly involve CP violating phases. To analyze how they appear in the amplitude, we first look into the generation of neutrino mass and mixing in this model. In the flavor basis for the neutrinos, the mass terms can be written in the form

$$\sum_{i,i'} \bar{\nu}_i \left( M_{ii'} \mathbf{P}_L + M_{i'i}^* \mathbf{P}_R \right) \nu_{i'}, \quad (43)$$

where the matrices appearing in the two chiral terms are related by the condition of hermiticity of their sum. Because of this relation, we can work entirely with just one part, say  $M$ , and call it the mass matrix. Diagonalization of this matrix was discussed in Eqs. (32) and (33). Once this part is diagonalized and the eigenbasis is determined for left- and right-chiral fields, the other term is automatically diagonal in the same basis.

It is to be noted that trilinear RPV couplings do not generate neutrino masses at the tree level. The masses are generated through 1-loop diagrams which look like the diagrams of Figs. 1 and 2 *without* the photon line. If we calculate these mass diagrams with neutrino flavor states on the outer lines and with vanishing external momentum, we obtain

$$M_{ii'} = \sum_{aA} m_{da} \left( \widehat{h}'_{iaA} h'_{i'aA} + \widehat{h}'_{i'aA} h'_{iaA} \right) J'_{aA}, \quad (44)$$

with the integral  $J'_{aA}$  defined as

$$J'_{aA} = i \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 - m_{da}^2][k^2 - \widetilde{M}_{\Delta A}^2]}, \quad (45)$$

and the couplings defined by

$$\begin{aligned}\widehat{h}'_{iaA} &= \sum_{jk} \lambda'_{ijk} V_{ja} \widehat{K}_{kA}^*, \\ h'_{iaA} &= \sum_{jk} \lambda'_{ijk} \delta_{ka} K_{jA},\end{aligned}\tag{46}$$

which are reminiscent of the couplings  $\widehat{\Lambda}$  and  $\Lambda$  in the neutrino flavor basis. The integral  $J'$  itself is logarithmically divergent, but we need to remember that the argument around Eq. (41) guarantees that  $M_{ii'}$  vanishes if all squarks are degenerate. Since the divergence is independent of all masses, it cancels anyway. Of course, this has to happen since the theory is renormalizable and there is no tree-level mass term.

In the simplified scenario advocated above where we neglect quark mixing and inter-generational squark mixing as well, the mass formula trivially reduces to

$$M_{ii'} = \sum_{jk} m_{dj} (\lambda'_{ijk} \lambda'_{i'kj} + \lambda'_{i'jk} \lambda'_{ikj}) \sin \theta_k \cos \theta_k (J'_{j,k+n} - J'_{jk}),\tag{47}$$

which is what we will use below.

## 4.2 Ansatz for a simplified analysis

Phases can enter the mass matrix from any of the couplings  $\lambda'_{ijk}$ . For three generations, there are 27 such couplings, and the phases are all independent. For illustrative purpose, we make a simplifying assumption that the phases can be written in the form

$$\lambda'_{ijk} = e^{i(\omega_i + \phi_j + \chi_k)} \left| \lambda'_{ijk} \right|, \quad \forall i, j, k.\tag{48}$$

Since we are considering leptonic processes, it would have been most natural to focus on the phases  $\omega_i$  only, which pertain to lepton doublets. However, it is easy to see that if these are the only RPV couplings, i.e., the couplings are of the form

$$\lambda'_{ijk} = e^{i\omega_i} \left| \lambda'_{ijk} \right|, \quad \forall i, j, k,\tag{49}$$

the model conserves CP. The reason is that in this case, we can absorb all phases of the  $\lambda'_{ijk}$ 's into the definition of the leptonic superfields  $L_i$ . The phases of these superfields will then reappear in the Yukawa couplings with the relevant Higgs doublet superfield, but those couplings can again be made real by readjusting the phases of the superfields  $E_j^c$ . Thus, at the end, no CP violating phase remains. If the phases denoted by  $\phi$  and  $\chi$  are non-zero, the possibility of CP violation remains, but the phases denoted by  $\omega$  continue to be irrelevant for the reason stated above.

So we drop  $\omega_i$  altogether. Eq. (48) now boils down to

$$\lambda'_{ijk} = e^{i(\phi_j + \chi_k)} \left| \lambda'_{ijk} \right|, \quad \forall i, j, k.\tag{50}$$

Through Eq. (47), this will introduce phases in the neutrino mass matrix. This mass matrix is diagonalized by the matrix  $U$  through Eq. (33), so that the phases in  $U$  are functions of the  $\phi$ 's and the  $\chi$ 's. The functional relation is, however, non-trivial for three generations, where there are two Majorana-type and one Dirac-type phases in the neutrino mixing matrix [18].

We, however, note from the expressions in Eqs. (39) and (40) that phases can enter the amplitude only through the quantities called  $B_{jk}$ . Clearly, the combinations occurring in Eq. (39) are very different from those occurring in the mass matrix, Eq. (47), because the two equations involve different integrals and different combinations of the couplings. This implies that radiative decay amplitudes and neutrinoless double beta decay can in principle provide independent probes of CP violating phases.

## 5 Conclusions

Our primary intention has been the derivation of the radiative Majorana neutrino decay amplitude in the  $R$ -parity violating supersymmetric framework, with a careful handling of the phases from different sources that enter the analysis. Our Eq. (25) is the most general expression for the radiative decay amplitude involving the  $\lambda'$  couplings. In the derivation, we have not taken recourse to any specific assumption regarding the masses and mixings of fermions and sfermions. We have relied only on the general field-theoretic features, highlighting some subtle aspects regarding the Majorana character of the neutrinos which have not been clearly dealt with in the literature. We have shown, even in this general framework, that the radiative decay amplitudes depend on the quark masses and on the mass-squared differences of squarks. Our general formula can be used for all sorts of different realizations of fermion and sfermion masses and mixings. We have provided an illustrative example in Sec. 4.2, where we have pointed out that radiative decay amplitudes and neutrinoless double beta decay can in principle provide independent probes of CP violating phases. We do not discuss the numerical implications of our result, because they are of the same order of magnitude as those obtained in earlier papers, e.g., in Refs. [7, 8].

As mentioned in the Introduction, we have not included the contributions from the  $\lambda$  couplings. In fact, it is trivial to include these contributions. The diagrams will now involve charged leptons and sleptons in the internal lines, instead of quarks and squarks. Thus the contributions can be directly adapted from the  $\lambda'$ -induced contributions by making suitable substitutions.

**Acknowledgements :** GB and HP acknowledge hospitality at CERN Theory Division as short term visitors during the initial stages of the work. HP was supported by US DOE under the grant DE-FG02-04ER41291. TJW acknowledges support from DOE grant DE-FG05-85ER40226, and the CERN Paid Associates program.

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